The Optimal Resource Allocation in Stochastic Activity Networks via Continuous Time Markov Chains

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Abstract

The problem we investigate deals with the optimal assignment of a resource to the activities of a stochastic project network. We seek to minimize the expected cost of the project, which we take as the sum of resource utilization costs and lateness cost, if the project is completed after a specified due date. Both costs are functions of the resource allocations to the activities with opposite responses to changes in allocation. We assume that the work content required by the activities follows an exponential distribution. The duration of each activity also follows an exponential distribution based on the degree of resource allocation. We construct a continuous time Markov chain (CTMC) model for the activity network and use the Phase-Type (PH-type) distribution to evaluate the project completion time. We developed a derivative-like descent algorithm for the optimization of the expected cost of the project. These quasi-derivatives lead to the selection of an activity to which we optimize the resource allocation. We use Fibonacci search over the interval of permissible allocations to the activity to determine the minimum expected cost. This iterative process of activity selection followed by changing the resource allocation is repeated until the expected cost is not significantly decreased. Finally, through extensive experimentation with a variety of projects of different structure and size, we show that this algorithm yields promising results in terms of both computation time and accuracy.

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